Innovative method to investigate how the spatial correlation of the pump beam affects the purity of polarization entangled states

Simone Cialdi,¹,²* Davide Brivio,³ Andrea Tabacchini,¹ Ali Mohammed Kadhim,² and Matteo G. A. Paris¹,⁴

¹Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milan, Italy
²INFN, Sezione di Milano, I-20133 Milan, Italy
³Institute of Laser for Postgraduate Studies, Baghdad University, P.O. Box 47314, Iraq
⁴CNISM, Udr Milano, I-20133 Milan, Italy

*Corresponding author: simone.cialdi@unimi.it

Received June 21, 2012; revised August 14, 2012; accepted August 17, 2012; posted August 21, 2012 (Doc. ID 171088); published September 18, 2012

We present an innovative method to address the relation between the purity of type-I polarization entangled states and the spatial properties of the pump laser beam. Our all-optical apparatus is based on a spatial light modulator, and it offers unprecedented control on the spatial phase function of the entangled states. In this way, we demonstrate quantitatively the relation between the purity of the generated state and the spatial field correlation function of the pump beam. © 2012 Optical Society of America

OCIS codes: 270.0270, 270.5585.

Spontaneous parametric downconversion (SPDC) is a crucial process in the development of quantum technology, and represents one of the most effective sources of entangled photon pairs and of single photons [1,2]. For these reasons, the spatial and the spectral properties of the downconverted beams have been extensively analyzed as a function of the coherence properties of the pump beam [3–6]. Less attention has been paid to the effect of the spatial properties of the pump on the purity of polarization entangled states, especially those generated with type-I parametric downconversion, since this may be revealed only by an accurate control of the phase profile of the output beams. In this Letter, we exploit an all-optical innovative method based on a spatial light modulator (SLM) to gain an unprecedented control on the spatial phase function of the generated entangled states and demonstrate experimentally the relation between the purity of the generated state and the field correlation function of the pump beam.

The downconverted state at the output of the two crystals, assuming that the spectra of the pump and of the parametric downconversion are quasi-monochromatic, may be written as

$$\Psi = \frac{1}{\sqrt{2}} \int d\theta_0 d\theta_1 \sin[\Delta_{k_\parallel}] \left[ F(\Delta_{k_\perp}) \right. \times \left. \left[ |H, \theta_0\rangle \langle H, \theta_1| + e^{i\Phi(\theta_0, \theta_1)} |V, \theta_0\rangle \langle V, \theta_1| \right] \right],$$

(1)

where $L$ is the crystal length and $|P, \theta\rangle$ denote a single-photon state with polarization $P = H, V$ emitted at angle $\theta$. $\Delta_{k_\parallel}$ and $\Delta_{k_\perp}$ are the shifts with respect to the phase-matching condition of the longitudinal and transverse momentum of the two photons. The sinc function comes from the integration along the longitudinal coordinate inside the crystals, and the function $F$ from the integration over the transverse coordinate: denoting by $A_p(x)$ the complex amplitude of the pump, we have $F(\Delta_{k_\parallel}) = \int dx A_p(x) e^{i\Delta_{k_\parallel} x}$. The phase term $\Phi(\theta_0, \theta_1)$ arises from the optical path of the two photons generated in the first crystal inside the second crystal, and from the spatial walk-off between the $H$ and the $V$ beams of the downconversion outside the crystals [7,8]. In general, we may write $\Phi(\theta_0, \theta_1) = \phi(\theta_0) + \phi(\theta_1) + \Phi_\theta$, where, up to first order, we have $\phi(\theta) \approx n^\theta k L / \cos(\theta_0 + \theta_\parallel - \kappa L) \tan[(\theta_0 + \theta_\parallel - \kappa L) / n^\theta] = \delta(\theta_0 + \theta_\parallel - \kappa L) \cos(\theta_0 + \theta_\parallel - \kappa L)$, where $n^\theta$ is the extraordinary index of refraction in the second crystal, $k = 2\pi / \lambda$, $\theta_0$ is the central angle, and $L$ is the crystal length. The term $\Phi_\theta$ represents the additional phase possibly added by any external optical component, e.g., the SLM. Since we employ noncollinear SPDC, we can address the two variables $\theta_0$ and $\theta_1$ independently by the different region of the SLM. The shifts $\Delta_{k_\parallel}$ and $\Delta_{k_\perp}$ are given by

$$\Delta_{k_\parallel} = k_p - k_s \cos[(\theta_0 + \theta_\parallel)/n^\theta] - k_i \cos((\theta_0 + \theta_\parallel)/n^\theta)$$

$$\times \Psi(\theta_0 + \theta_\parallel) = k_\theta \theta_\perp,$$

$$\Delta_{k_\perp} = k_s \sin[(\theta_0 + \theta_\parallel)/n^\theta] - k_i \sin((\theta_0 + \theta_\parallel)/n^\theta)$$

$$\times \Psi(\theta_0 + \theta_\parallel) = k_\theta \theta_\perp, \quad (2)$$

where $\theta_+ = \theta_0 + \theta_1$ and $\theta_- = \theta_0 - \theta_1$, and $n^\theta$ is the ordinary index of refraction. Using the new variables, the overall phase function rewrites as

$$\Phi(\theta_+, \theta_-) = \phi_0 + \phi_\theta \theta_+ + \Phi_\theta.$$

The purity of the state, which in this case equals the visibility, may be written as

$$p = \int d\theta_+ d\theta_- |\sin(\gamma \theta_+)|^2 |F(k \theta_+)|^2 \cos \Phi(\theta_+, \theta_-),$$

where $\gamma = \frac{1}{2} k_\theta L$. The normalization condition is given by

$$\int d\theta_+ d\theta_- |\sin(\gamma \theta_+)|^2 |F(k \theta_+)|^2 = 1.$$
Two cases are of special interest: if \( \Phi_a = -\phi_0 \) the purity does not depend on \( F \) and we obtain the case that is usually described in the literature \([7,8]\). On the other hand, upon imposing \( \Phi_a = -\phi_0 - \alpha_0 \theta_+ + \beta \theta \), one obtains \( \Phi = \beta \theta ; \) i.e., the purity is now a function of \( F \). In this second case, using the Wiener–Khinchin theorem, we have

\[
p = \int \, d\theta_p \, |F(k\theta_p)|^2 \cos \beta \theta_p \propto \left( A_p^*(x + \frac{\beta}{K}) A_p(x) \right)_x,
\]

i.e., the purity of the state is proportional to the spatial field correlation function of the pump beam.

The experimental setup is shown in Fig. 1. A linearly polarized cw 405 nm diode laser (Newport LQC405–40P) passes through two cylindrical lenses, which compensate beam astigmatism, then a spatial filter composed by two lenses and a pin-hole in the Fourier plane to obtain a Gaussian profile by removing the multimode spatial structure of the laser pump, and finally a telescopic system prepares a beam with the proper beam radius and divergence. A couple of 1 mm beta-barium borate crystals, cut for type-I downconversion, with optical axis aligned in perpendicular planes, are used as a source of polarization and momentum entangled photon pairs with \( \theta_0 = 3^\circ \). In order to match the above theoretical model, we use a compensation crystal on the pump, which removes the delay time between the vertical and horizontal polarization \([8,9]\), and put a 10 nm interference filter on the signal path, in order to reduce the spectral width of the generated radiation. An SLM, which is a liquid crystal phase mask (64 \times 10 \text{ mm}) divided in 640 horizontal pixels, each \( d = 100 \mu \text{m} \) wide, is set before the detectors in order to introduce the spatial phase function at 310 mm from the generating crystals \([7]\). We also place a window of 5 mm in front of the couplers of the detectors. A cylindrical lens is placed immediately after the two generating crystals, whereas a camera sets a focal distance (1 m) to obtain the square modulus of the Fourier transform of the pump.

In order to complete the theoretical model, we have to take into account the fact that the spatial coupling is not flat, but rather has a Gaussian profile with an FWHM of about 5 mm. We thus insert this function when tracing out the spatial degrees of freedom in order to obtain the polarization state and its purity. In addition, there are some elements that introduce decoherence not compensable with the SLM; these are the gaps between the pixels of the SLM (3 \( \mu \text{m} \)), the imperfect compensation of the delay time, the spectral effects of the parametric downconversion, and the imperfect superposition between the amplitudes generated by the two crystals. In order to include these effects in the model, we write

\[
\rho = \rho_\text{mix} = \rho_p \rho_\text{p}^\dag \left( 1 - m \right) + \left( 1 - m \rho_p \right) \rho_\text{mix}.
\]


![Fig. 1.](image)

Fig. 1. (Color online) Schematic diagram of the experimental setup. The dashed part is used to measure the Fourier transform of the pump beam, and it is not present during measurements on the PDC output.

![Fig. 2.](image)

Fig. 2. (Color online) Purity (visibility) as a function of \( \alpha \) for a beam with spot of 220 \( \mu \text{m} \). The phase function imposed by the SLM is given by \( \Phi_a = -\phi_0 - \alpha_0 \theta_+ \). Error bars on the experimental values are within the points. The solid black line is the theoretical prediction.
the beam is divergent (by few milliradian). Now the visibility is a Gaussian function with a smaller width. In the last example, we place a grid with a step of 100 μm in front of the two generating crystals. In the spatial profile, we obtain two peaks, and this corresponds to a revival in the visibility after a collapse. In fact, the pump degrees of freedom represent a noisy environment for the polarization degrees, and thus upon modifying the spatial pump profile we are modifying its noise properties. The correlation properties of the pump induce spatial memory effects for the polarization degrees of freedom [10, 11], which correspond to a non-Markovian dynamics. The small shift in the bottom right picture (about 0.2 rad/mrad) is probably due to an imperfect compensation (of about 30 μm) of the spatial walk-off between the H and V polarization of the pump beam.

In conclusion, we have demonstrated the quantitative relation between the purity of type-I polarization entangled states and the spatial properties of the pump. In order to obtain this result, we exploited the unprecedented control of the spatial phase function of the generated states that is achievable by the use of a SLM on noncollinear SPDC. Our method may be used for entanglement engineering [12] and purification [7], and it paves the way for investigating fundamental effects in non-Markovian open systems [13].

We acknowledge support from the Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR) project FIRB LiCHIS-RBFR10YQ3H.

Fig. 3. Spatial field correlations of the pump beam and purity of the entangled output in three relevant cases: (first row) collimated pump beam of 220 μm, (second row) divergent pump beam of 220 μm, and (third row) pump beam with two peaks. We report the spatial profile of the pump (left column), its Fourier transform (right column), and the visibility as a function of β (right column). The phase function imposed by the SLM is given by $\Phi_\alpha = -\phi_0 - \alpha_0 \theta_+ + \beta \theta_-$.

References